String cosmology from a novel renormalization perspective: perturbing the fixed point

Jean Alexandre and **Nikolaos E. Mavromatos**

Department of Physics, King's College London, London WC2R 2LS, England

Abstract

We study small perturbations around the exactly marginal time-dependent string configuration of [1], and demonstrate the lack of the appropriate linearization. This implies that this configuration is an isolated fixed point of the α' flow in the pertinent space of theories.

In [1], the authors were interested in a time-dependent configuration of the bosonic string, relevant to the description of a spatially-flat Robertson Walker Universe, with metric $ds^2 = -dt^2 + a^2(t)(d\vec{x})^2$, where t is the time in the Einstein frame, and $a(t)$ is the scale factor. It was shown that the following time-dependent configuration

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{\gamma} \left\{ \gamma^{ab} \frac{\kappa_0}{(X^0)^2} \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu + \alpha' R^{(2)} \phi_0 \ln(X^0) \right\}, \qquad (1)$$

where κ_0 and ϕ_0 are constants, does not get renormalized after quantization. It was then conjectured that it satisfies Weyl invariance conditions in a non-perturbative way, for any target space dimension D. The corresponding scale factor was then shown to be a power law

$$a(t) \propto t^{1 + \frac{D-2}{2\phi_0}}, \qquad (2)$$

such that, if the following relation holds

$$D = 2 - 2\phi_0, \qquad (3)$$

the target space is static and flat (Minkowski Universe).

Our aim in this note is to check on the stability properties of the configuration (1) under small perturbations (linearization, to be defined below). Specifically, we shall consider small perturbations around the configuration (1), with metric $g_{\mu\nu}$ and dilaton ϕ such that

$$
\begin{aligned}
g_{\mu\nu}(X^0) &= \kappa(X^0)\eta_{\mu\nu} = \frac{\kappa_0}{(X^0)^2}\eta_{\mu\nu}\left(1 + \xi(X^0)\right) \\
\phi'(X^0) &= \frac{\phi_0}{X^0}\left(1 + \varepsilon(X^0)\right),
\end{aligned} \qquad (4)
$$

1

where a prime denotes a derivative with respect to X^0 and $\xi << 1$, $\varepsilon << 1$. For this configuration, the beta functions, whose vanishing would guarantee world sheet Weyl invariance, are to the lowest order in α' [1],

$$
\begin{aligned}
\beta_{00}^g &= -(D-1)\left(\frac{\kappa'}{2\kappa}\right)' + 2\phi'' - \frac{\kappa'}{\kappa}\phi' + \mathcal{O}(\alpha') \\
\beta_{ij}^g &= \delta_{ij}\left\{ \left(\frac{\kappa'}{2\kappa}\right)' + (D-2)\left(\frac{\kappa'}{2\kappa}\right)^2 - \frac{\kappa'}{\kappa}\phi' \right\} + \mathcal{O}(\alpha') \\
\beta^\phi &= \frac{D-26}{6\alpha'} - \frac{\phi''}{2\kappa} - \frac{(D-2)}{4}\frac{\kappa'}{\kappa^2}\phi' + \frac{(\phi')^2}{\kappa} + \mathcal{O}(\alpha').
\end{aligned}
\tag{5}
$$

Substituting the perturbation (4) and keeping only the linear terms in ξ and ε, leads to

$$
\begin{aligned}
\beta_{00}^g &= -\frac{D-1}{(X^0)^2} + \frac{1}{(X^0)^2}\left\{ -\frac{D-1}{2}(X^0)^2\xi'' + \phi_0 X^0(2\varepsilon' - \xi') \right\} + \mathcal{O}(\alpha') \\
\beta_{ij}^g &= \delta_{ij}\frac{D-1+2\phi_0}{(X^0)^2} + \frac{\delta_{ij}}{(X^0)^2}\left\{ \frac{(X^0)^2}{2}\xi'' - (D-2)X^0\xi' + \phi_0(2\varepsilon - X^0\xi') \right\} + \mathcal{O}(\alpha') \\
\beta^\phi &= \frac{D-26}{6\alpha'} + \frac{\phi_0}{2\kappa_0}(D-1+2\phi_0) \\
&\quad + \frac{\phi_0}{2\kappa_0}\left\{ -X^0\left(\varepsilon' + \frac{D-2}{2}\xi'\right) + (D-1)(\varepsilon - \xi) + 2\phi_0(2\varepsilon - \xi) \right\} + \mathcal{O}(\alpha').
\end{aligned}
\tag{6}
$$

We are looking for solutions ξ, ε which, at this order in α', do not change the homogeneity in X^0 of the beta functions, and therefore satisfy

$$
\begin{aligned}
0 &= -\frac{D-1}{2}(X^0)^2\xi'' + \phi_0 X^0(2\varepsilon' - \xi') \\
0 &= \frac{(X^0)^2}{2}\xi'' - (D-2+\phi_0)X^0\xi' + 2\phi_0\varepsilon \\
0 &= -X^0\left(\varepsilon' + \frac{D-2}{2}\xi'\right) + (D-1+4\phi_0)\varepsilon - (D-1+2\phi_0)\xi
\end{aligned}
\tag{7}
$$

In what follows, we are interested in a Minkowski target space, and consider therefore the situation where $D = 2 - 2\phi_0$, such that

$$
\begin{aligned}
0 &= \left(\phi_0 - \frac{1}{2}\right)(X^0)^2\xi'' + \phi_0 X^0(2\varepsilon' - \xi') \\
0 &= \frac{(X^0)^2}{2}\xi'' + \phi_0 X^0\xi' + 2\phi_0\varepsilon \\
0 &= -X^0\left(\varepsilon' - \phi_0\xi'\right) + (1+2\phi_0)\varepsilon - \xi
\end{aligned}
\tag{8}
$$

The structure of these equations shows that the only possible solution is a power law for ε and ξ. We then assume the following X^0-dependence, consistent with our linearization

procedure,

$$\varepsilon = \frac{A}{(X^0)^a} \qquad \xi = \frac{B}{(X^0)^a}, \tag{9}$$

where A, B, a are constants to be determined. Using the first two of the equations (8), one arrives at

$$0 = 4\phi_0 + a^2 - 1. \tag{10}$$

And then, using the last two of equations (8), one obtains

$$1 + a^2 = 0, \tag{11}$$

which has no real solution.

We conclude that no linearization is possible around the Minkowski Universe described by the exactly marginal configuration (1), thereby suggesting that the latter is an isolated fixed point of the α' flow in the pertinent space of quantum theories.

References

[1] J. Alexandre, J. Ellis and N. E. Mavromatos, JHEP **0612** (2006) 071 [arXiv:hep-th/0610072].